

Optimally integrating renewables

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Uncertainty of renewable power

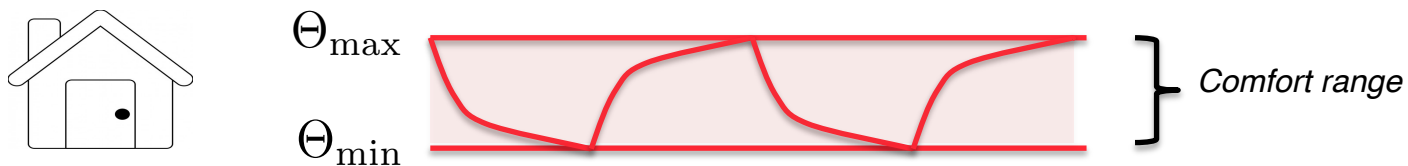
- ◆ Wind power is stochastic, not dispatchable



- ◆ How to integrate wind?

Demand response

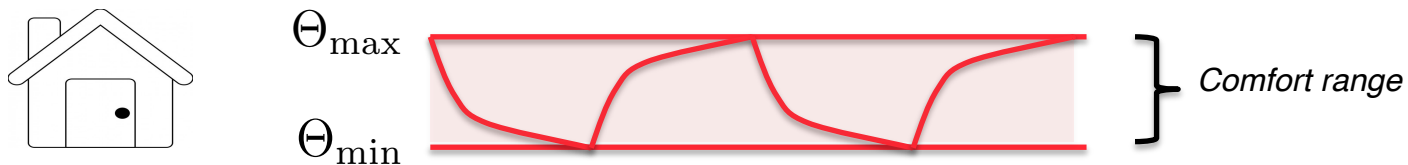
- ◆ Adjust *demand* to match supply
- ◆ Some loads can be switched off for a while without being noticed
 - E.g., Air conditioners under thermostatic control



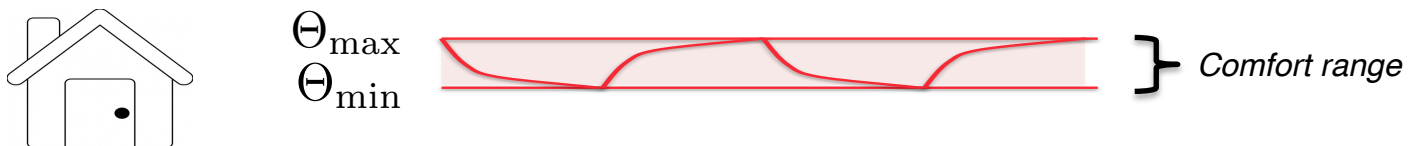
- ◆ Inertial thermal loads can absorb fluctuations in available wind power

Flexibility of load requirements

- ◆ Amount of demand response will depend on how flexible the loads are with respect to their requirements
- ◆ More demand response possible

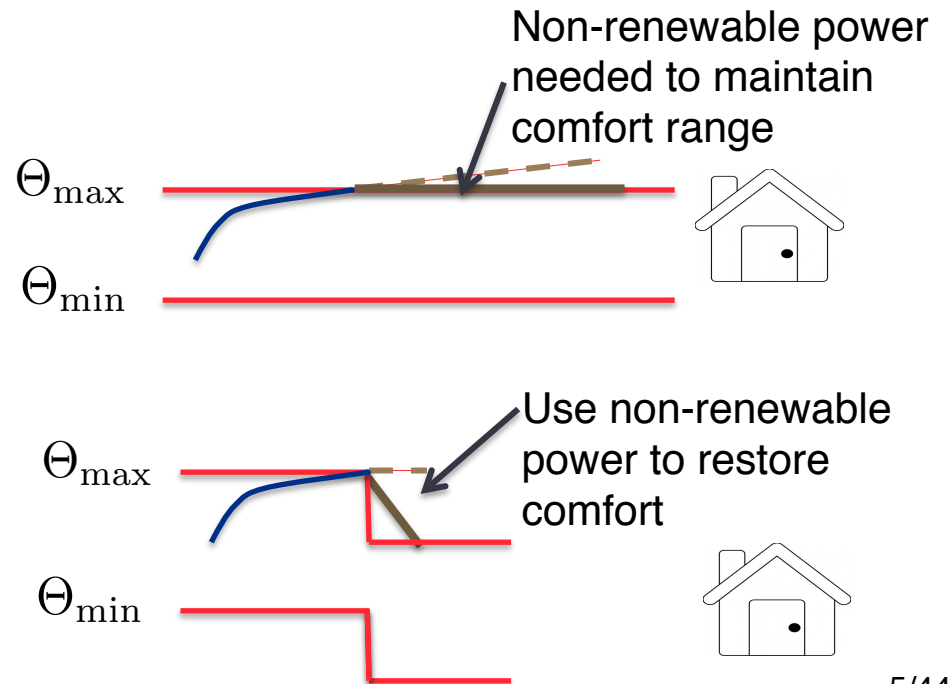


- ◆ Lesser scope for demand response



Renewable power is not enough to fully satisfy load requirements

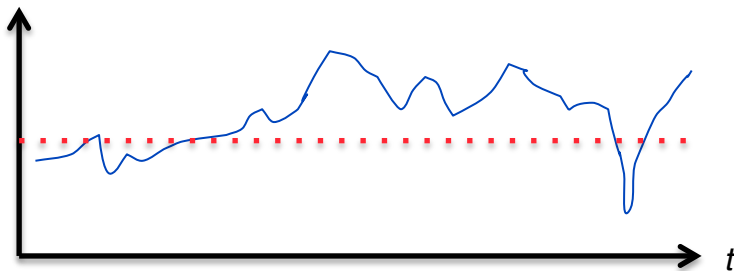
- ◆ Renewables can help *reduce* need for non-renewables
- ◆ However, they *cannot eliminate* need for non-renewables
- ◆ Non-renewables still required
 - When wind stops blowing
 - After sudden comfort-setting change



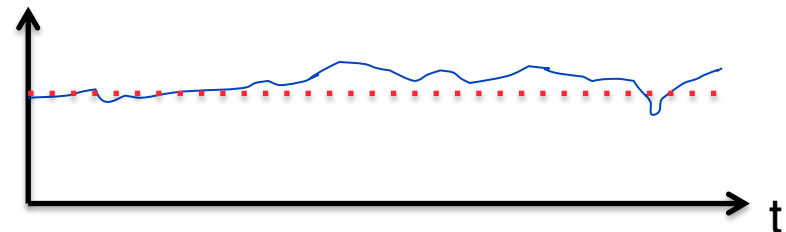
Reduce peak-to-average non-renewable power generation

- ◆ Non-renewables still required
- ◆ Need to reduce peak-to-average of non-renewable power

More variability



Less variability



- Reduce expensive spinning/other reserves, capital, etc

Concavity and desynchronization

A stochastic control problem

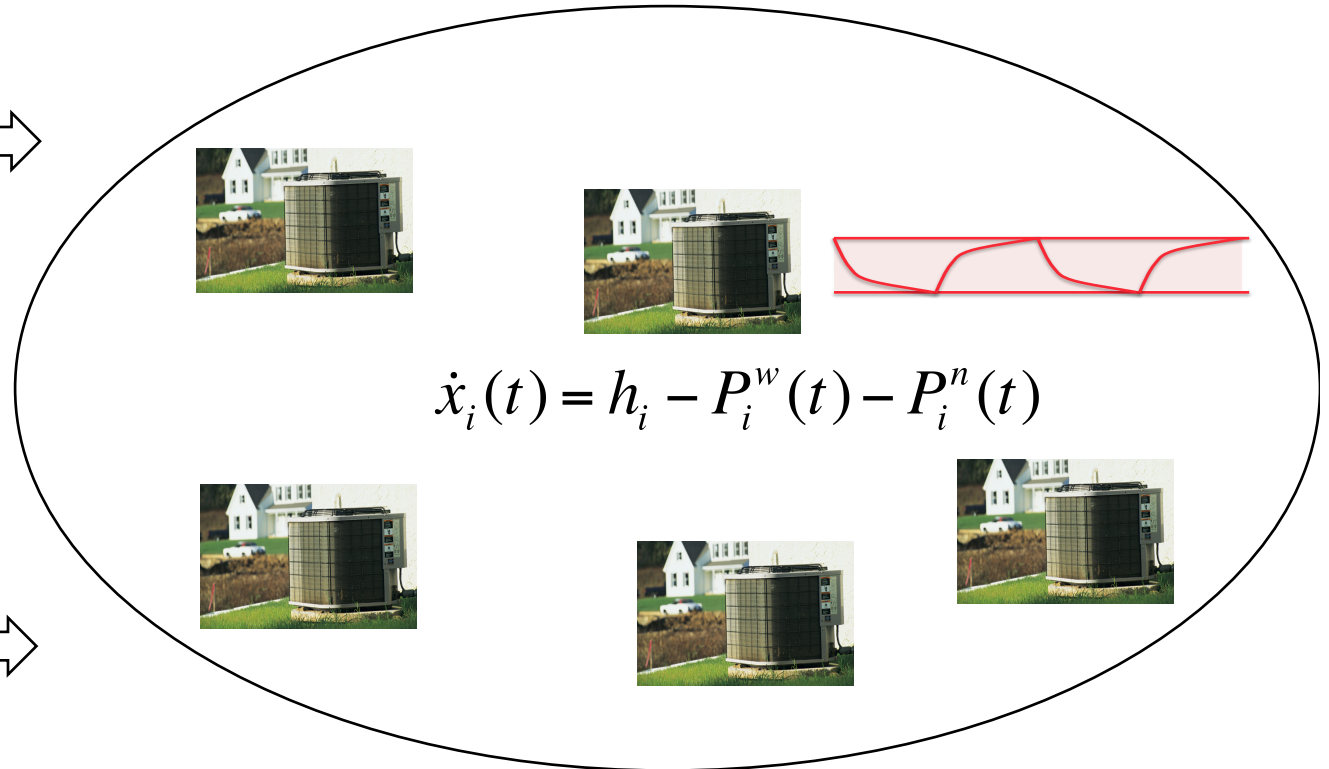
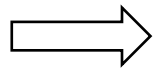
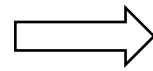
◆ Collection of loads



$P_{wind}(t)$



$P_{non-renewable}(t)$



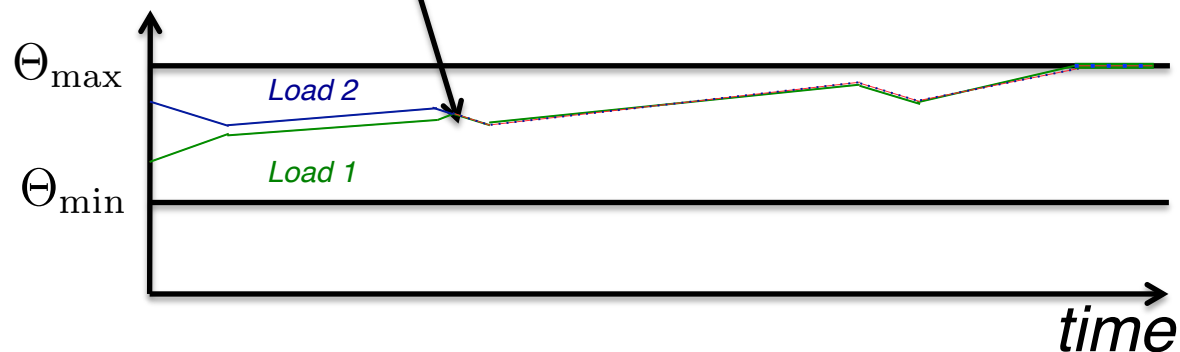
Stochastic control model

- ◆ Wind process $\sum P_i^w(t) \sim \text{Markov process}$
- ◆ Temperature dynamics $\dot{x}_i(t) = h_i - P_i^w(t) - P_i^n(t)$
- ◆ Non-renewable power $P_i^n(t) \geq 0$
- ◆ Temperature constraint $x_i(t) \in [\Theta_{\min}, \Theta_{\max}], \forall i$
- ◆ Quadratic cost to reduce variability $\lim_{T \rightarrow \infty} \frac{1}{T} \int_0^T [\sum_i P_i^n(t)]^2 dt$

Optimal solution: Synchronization

- ◆ Theorem: The optimal policy synchronizes loads

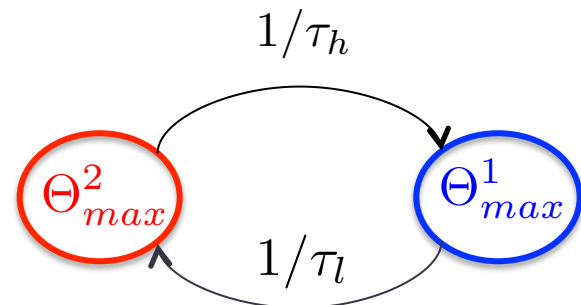
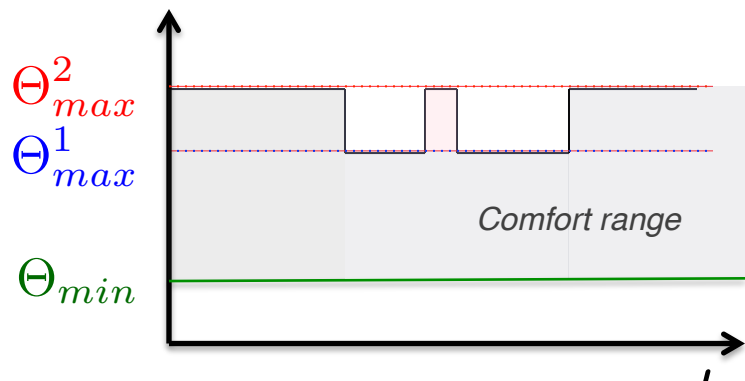
Loads will remain synchronized after this time instant



- ◆ Is there some modification in the model or cost function which leads to de-synchronization ?

Stochastic model for Θ_{max}

- ◆ Suppose users occasionally change Θ_{max} settings at the same time
 - E.g. Super Bowl Sundays @ game time
- ◆ Model changes in Θ_{max} as a two state Markov process



Resulting stochastic control problem

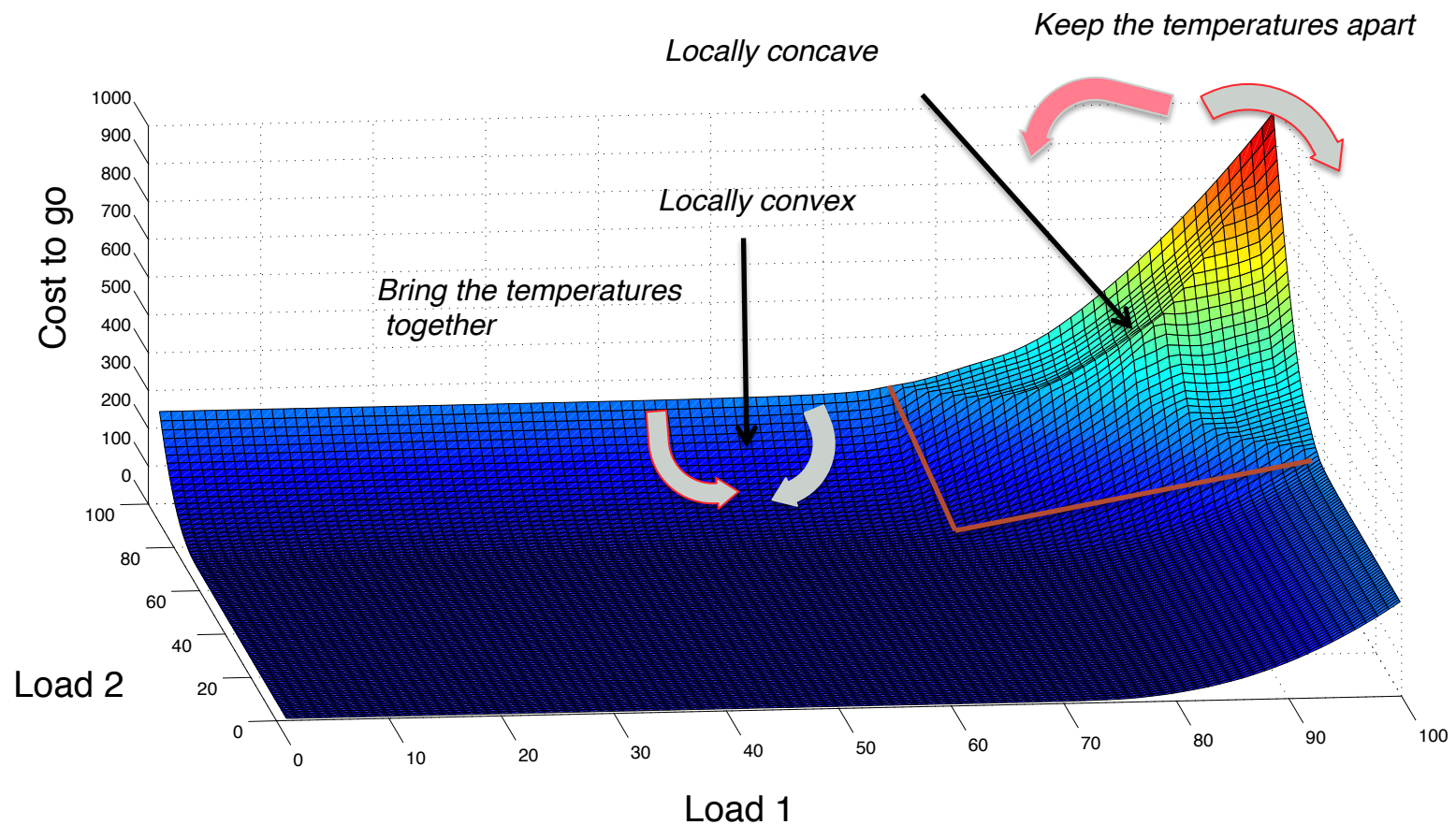
- ◆ Wind process: $\sum P_i^w(t) \sim \text{Markov process}$
- ◆ Temperature dynamics: $\dot{x}_i(t) = h_i - P_i^w(t) - P_i^n(t)$
- ◆ Non-renewable power $P_i^n(t) \geq 0$
- ◆ Stochastic comfort level $\Theta_{max}(t) \sim \text{Markov process}$, $\Theta_{max}(t) \in \{\Theta_{max}^1, \Theta_{max}^2\}$
- ◆ Temperature constraint: $x_i(t) \in [\Theta_{min}, \Theta_{max}^2], \forall i$
- ◆ Maximum cooling rate: $P_i^n(t) = M$ If $x_i(t) > \Theta_{max}(t)$
- ◆ Quadratic cost: $\lim_{T \rightarrow \infty} \frac{1}{T} \int_0^T [\sum_i P_i^n(t)]^2 dt$

HJB equation and optimal solution

- Cost to go function $V^{ij}(x, t) := \min_{P_i^n, P_i^w \in U} \mathbb{E} \left[\int_t^T (P_1^n + P_2^n)^2 | w(t) = i, th(t) = j, x(t) = x \right]$
- HJB equation
$$\min_{P_1^n, P_2^n \in U} \{ (P_1^n + P_2^n)^2 - \frac{\partial V^{ij}}{\partial x_1} P_1^n - \frac{\partial V^{ij}}{\partial x_2} P_2^n \} - \max_{P_1^w, P_2^w \in U} \{ \frac{\partial V^{ij}}{\partial x_1} P_1^w + \frac{\partial V^{ij}}{\partial x_2} P_2^w \} \chi_{\{i=1\}} \\ = q_{ii'}(V^{ij} - V^{i'j}) + q_{jj'}(V^{ij} - V^{ij'}) - h_i(V_{x_1}^{ij} + V_{x_2}^{ij}) - \dot{V}^{ij}$$
- Optimal Solution
$$(\dot{P}_1^w(\vec{x}, j), \dot{P}_2^w(\vec{x}, j)) = \begin{cases} (W, 0) & \text{if } \frac{\partial V_{1j}^*}{\partial x_1} > \frac{\partial V_{1j}^*}{\partial x_2} \\ (0, W) & \text{if } \frac{\partial V_{1j}^*}{\partial x_1} < \frac{\partial V_{1j}^*}{\partial x_2} \end{cases}$$

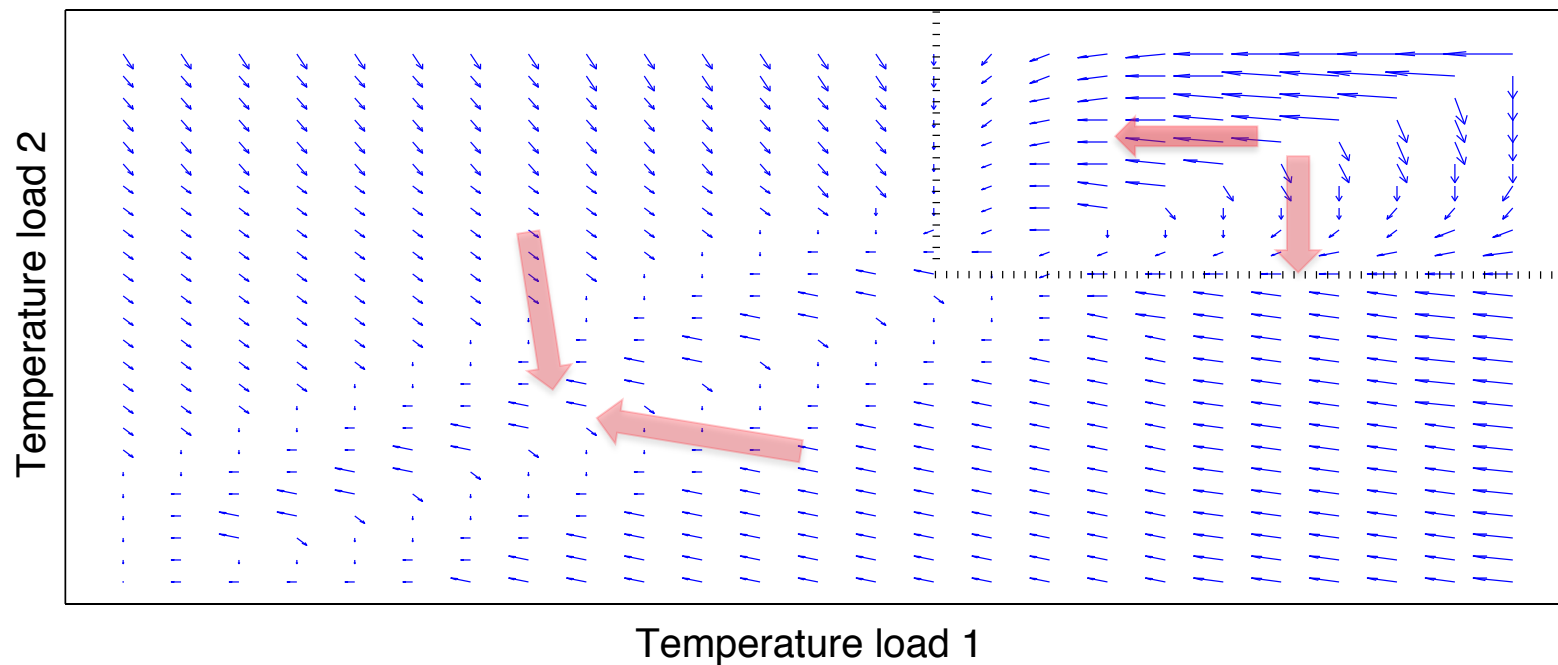
$$(\dot{P}_1^n(\vec{x}, i, j), \dot{P}_2^n(\vec{x}, i, j)) = \begin{cases} (\frac{1}{2} \frac{\partial V_{ij}^*}{\partial x_1}(\vec{x}), 0) & \text{if } \frac{\partial V_{ij}^*}{\partial x_1} > \frac{\partial V_{ij}^*}{\partial x_2} \\ (0, \frac{1}{2} \frac{\partial V_{ij}^*}{\partial x_2}(\vec{x})) & \text{if } \frac{\partial V_{ij}^*}{\partial x_1} < \frac{\partial V_{ij}^*}{\partial x_2} \\ (\frac{1}{2} \frac{\partial V_{ij}^*}{\partial x_1}(\vec{x}), \frac{1}{2} \frac{\partial V_{ij}^*}{\partial x_2}(\vec{x})) & \text{if } \frac{\partial V_{ij}^*}{\partial x_1} = \frac{\partial V_{ij}^*}{\partial x_2} \end{cases}$$
- Optimal power allocation depends upon $\frac{\partial V_{ij}^*}{\partial x_1} \leq \frac{\partial V_{ij}^*}{\partial x_2}$ when $x_1 \leq x_2$

Local concavity in stochastic Θ_{\max} variational model



Optimal solution for stochastic Θ_{\max} variation model

◆ Nature of the optimal solution

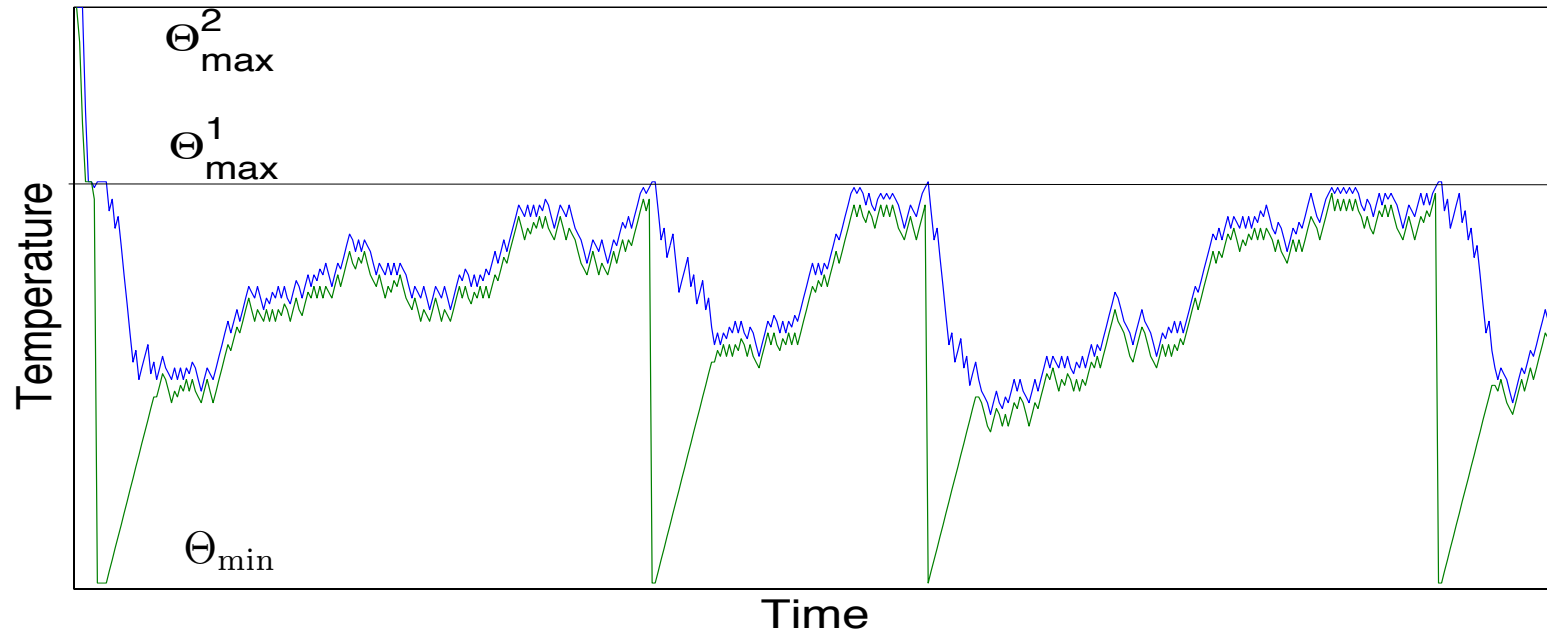


Vector field of temperature changes

- De-synchronization at high temperatures
- Re-synchronization at low temperatures

De-synchronization/Re-synchronization in solution

- ◆ It is optimal to separate at high temperatures



- Hedges against the future eventuality that the thermostats are switched low

Issues in designing an architecture and solution for demand response

Need for demand side and supply side information exchange

- ◆ Loads need to know *when* to invoke demand response
- ◆ Supply side needs to know *how much* demand response will provide
- ◆ Need for two-way communication between demand side and supply side
 - Volume of data
 - Delay requirements of data

Need to respect privacy

- ◆ How to control demand without intrusive sensing of temperatures of homes?

Need to reduce communication requirements

- ◆ How to minimize communication requirements for measurements and actuation signals?

Challenges

◆ Goals

- Maximize utilization of renewable energy
- Minimize variability of non-renewable power required
- Respect comfort constraints of homes

◆ Architecture

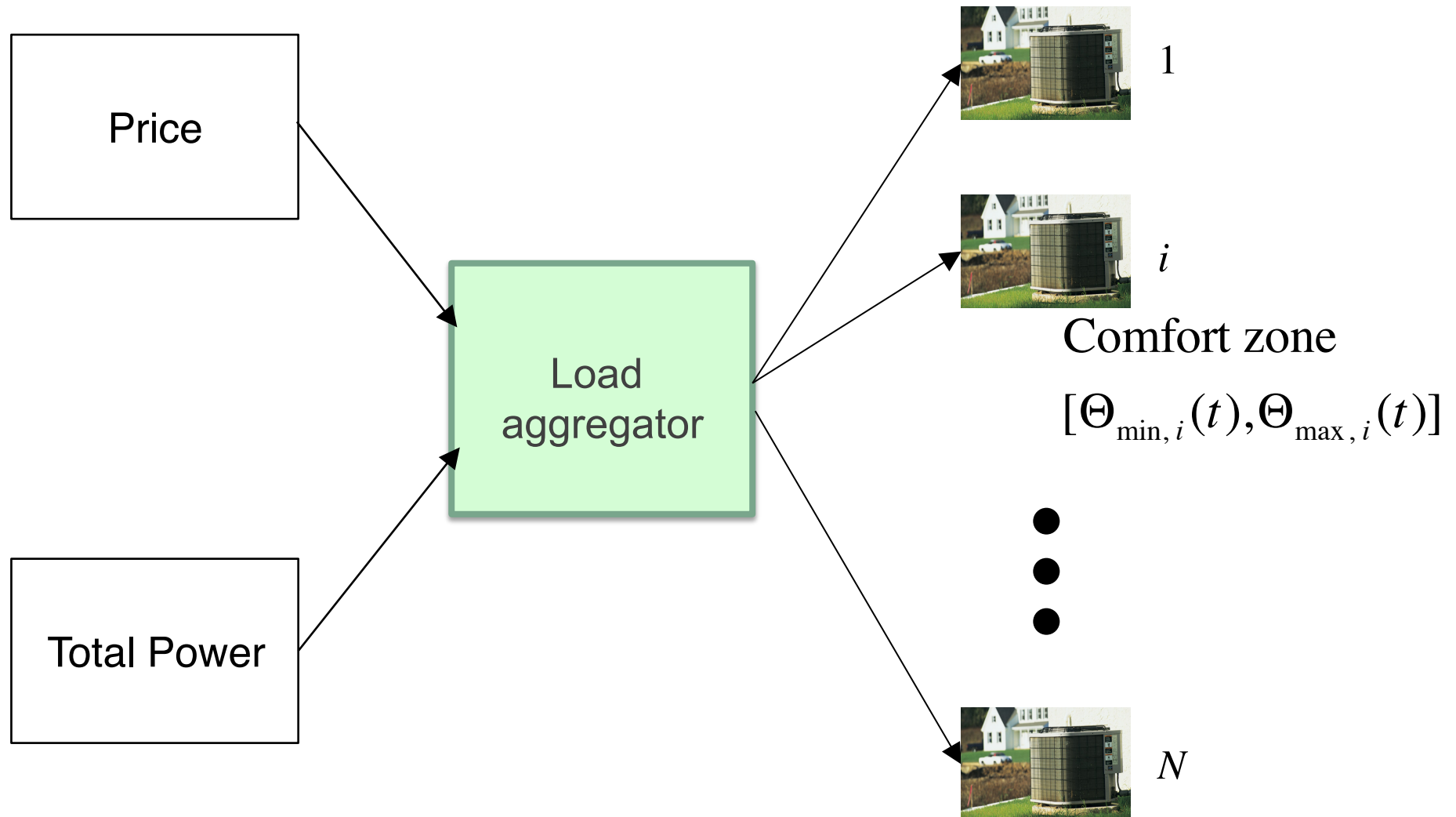
- How to achieve demand *pooling*?
- Respect privacy: No intrusive sensing
- Minimize communication requirements
 - » Volume and latency of data

◆ Solution

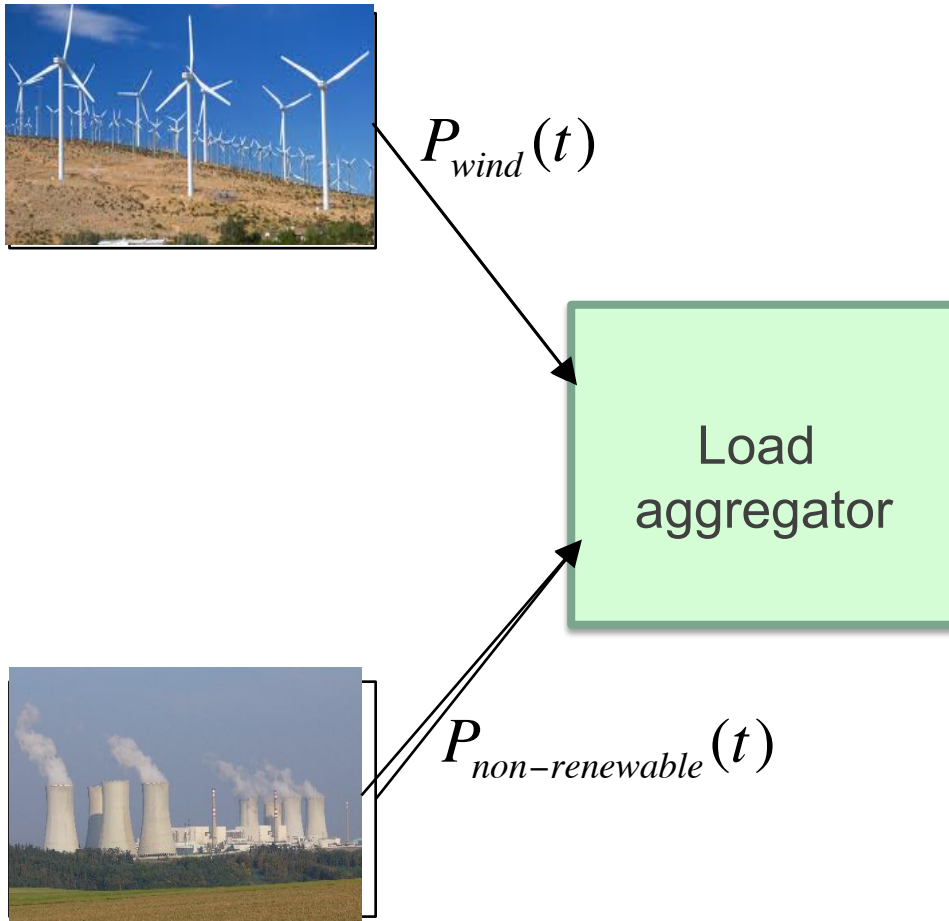
- “Optimal” – efficient in some sense
- Computationally tractable for large number of homes

Architecture of the solution

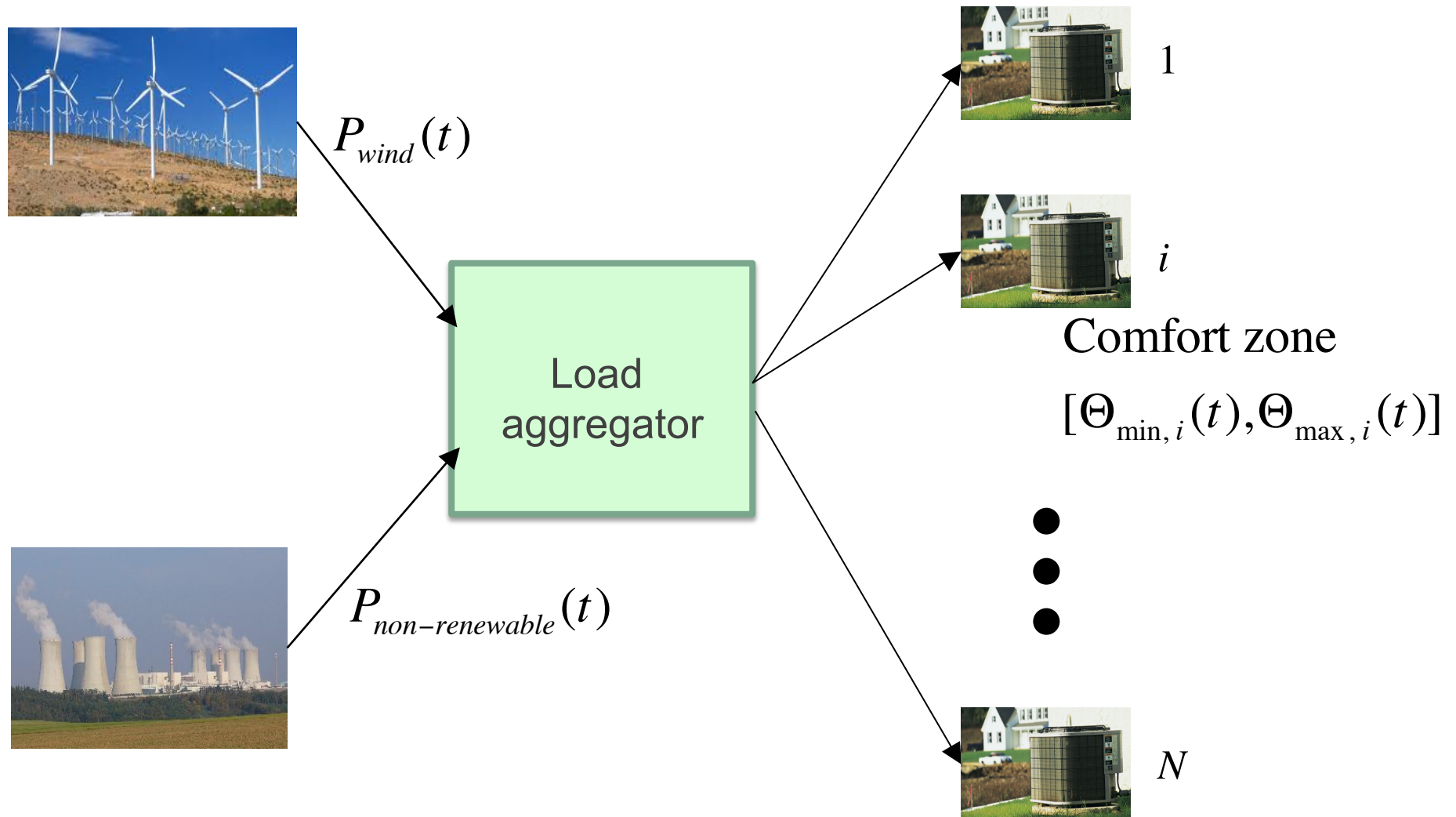
Load aggregator: Price based aggregation



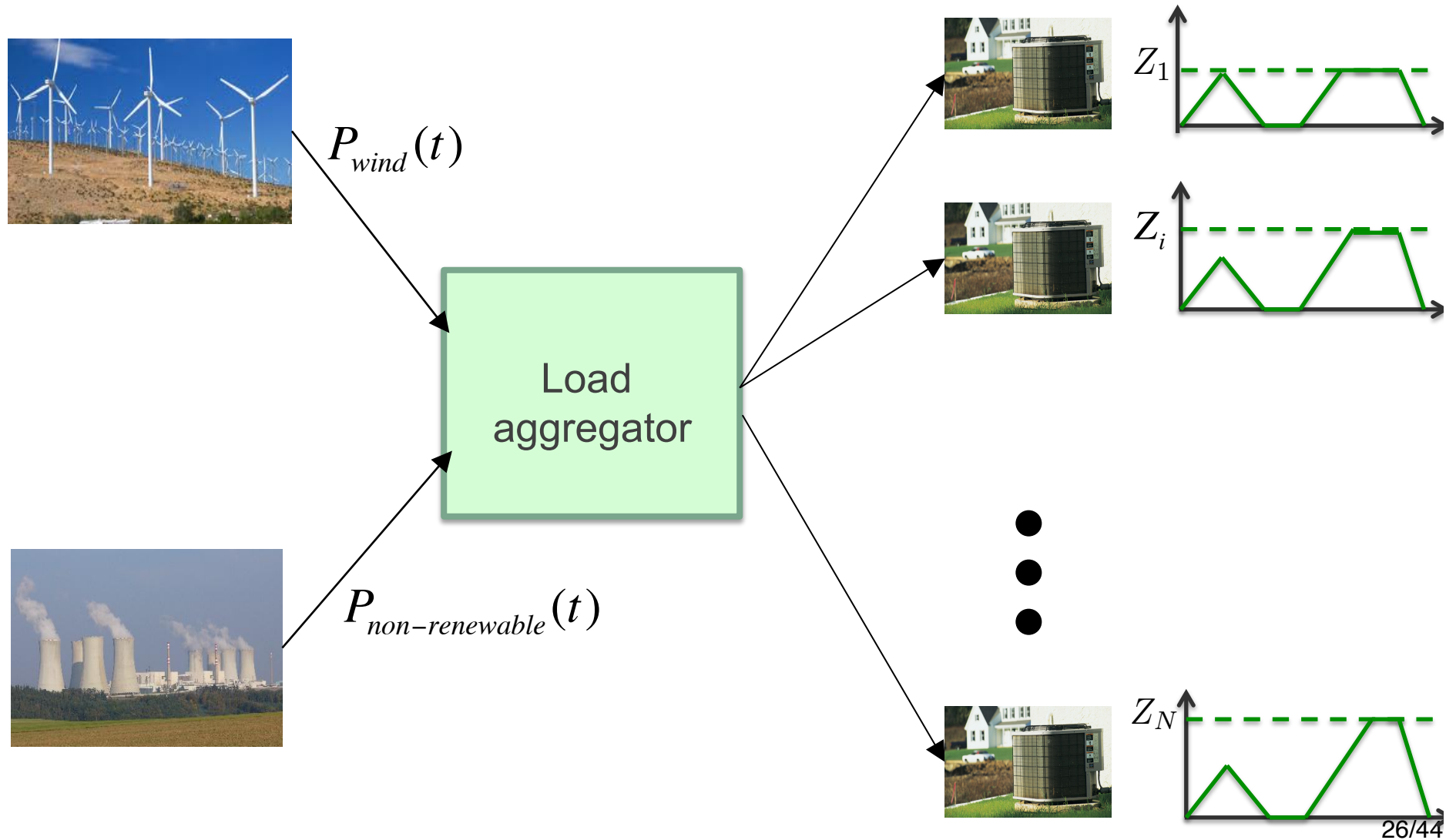
Load aggregator: Price based aggregation



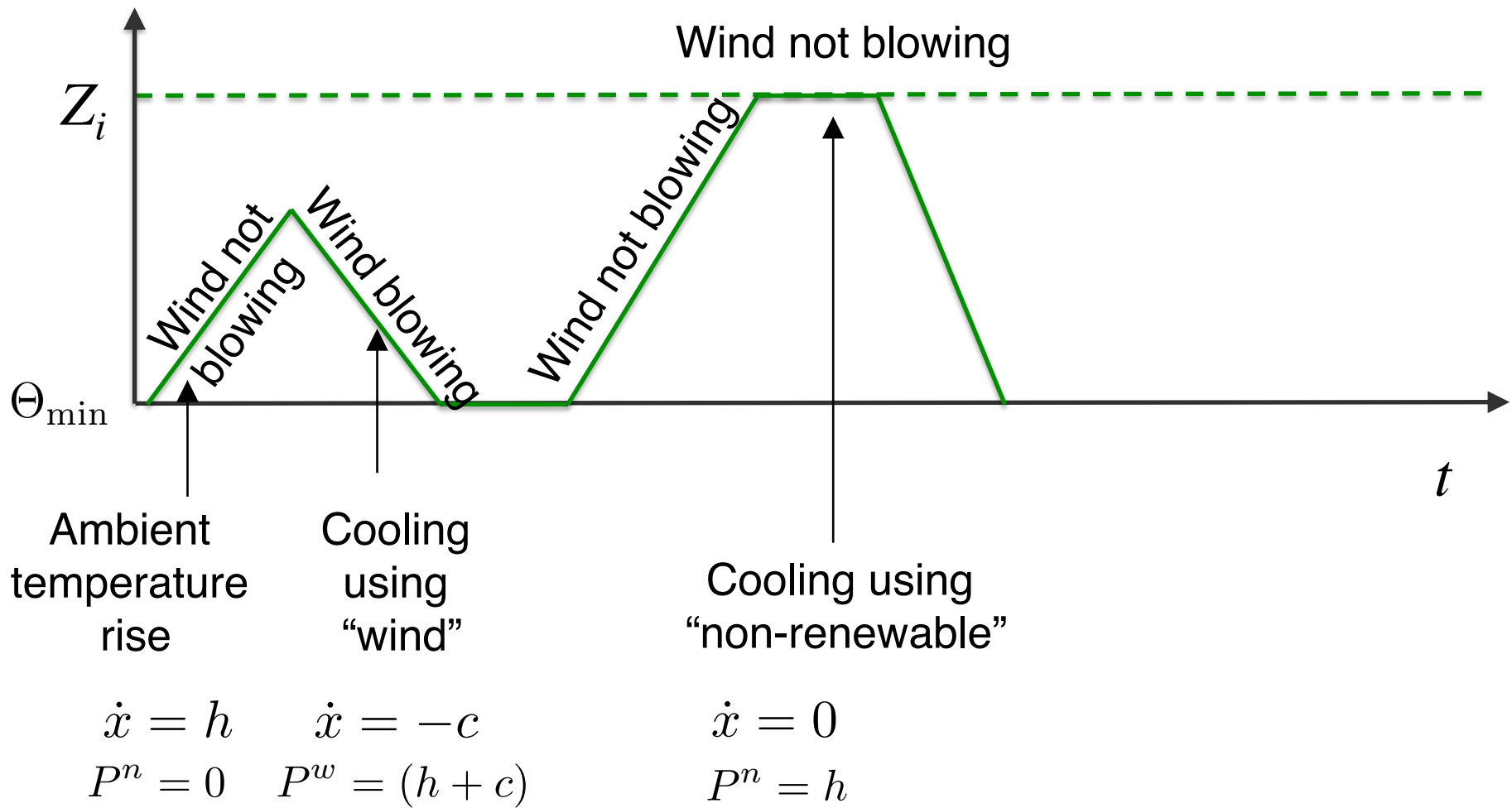
Load aggregator: Microgrid with renewable energy supply



Thermostatic control with set points Z_i

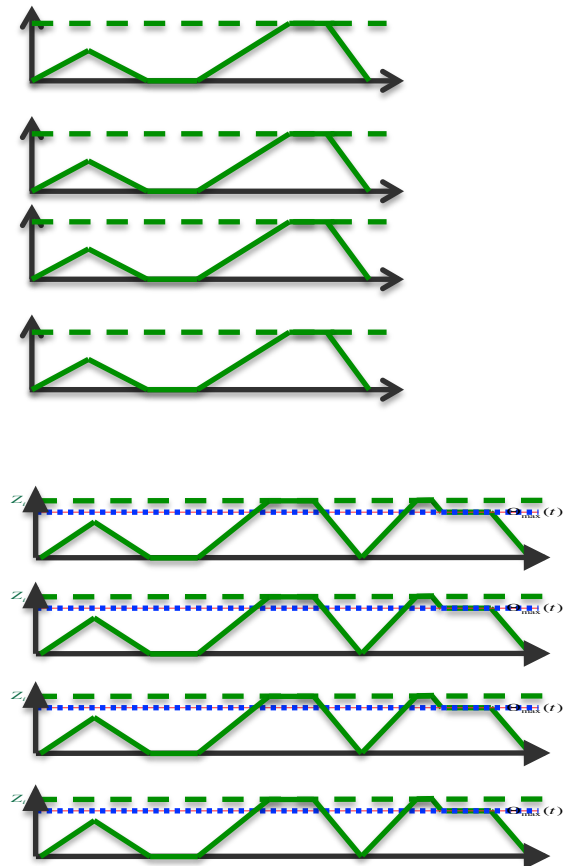


Thermostatic set-point based control policy



Problem: Synchronization of demand response

- ◆ Optimal solution: All users behave alike
- ◆ Loads synchronize and move in lock-step
- ◆ Robustness problem: Suppose users change comfort level settings at same time
 - Super bowl Sundays @ game time
- ◆ Demand suddenly rises, causing large peak in nonrenewable power required
 - Model cost as $\lim_{T \rightarrow \infty} \frac{1}{T} \int_0^T \left(P^n(t) \right)^2 dt$

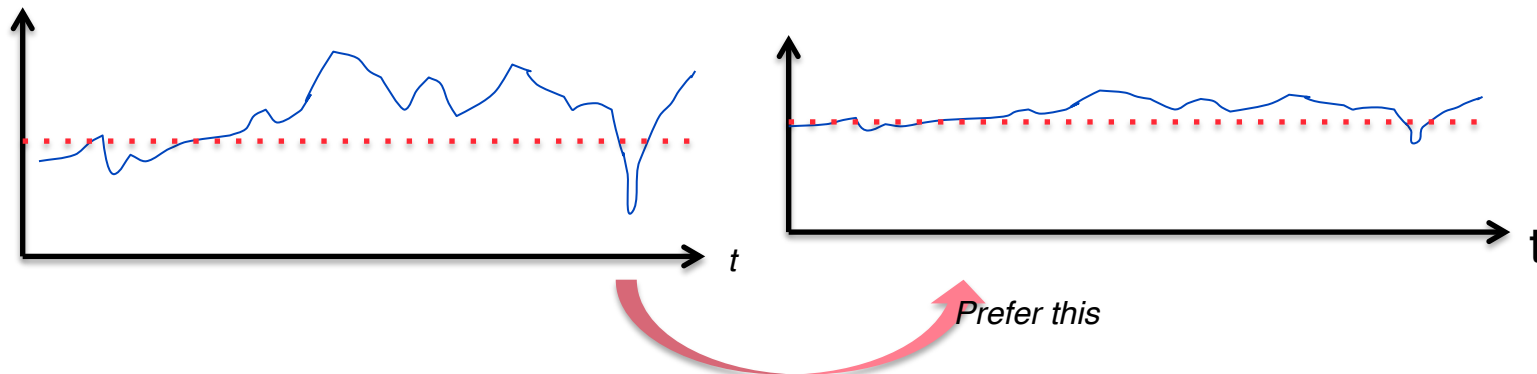


Reduce peak-to-average ratio of non-renewable power

- ◆ Low variability in non-renewable power consumption is desired

*More variability
Higher Quadratic cost*

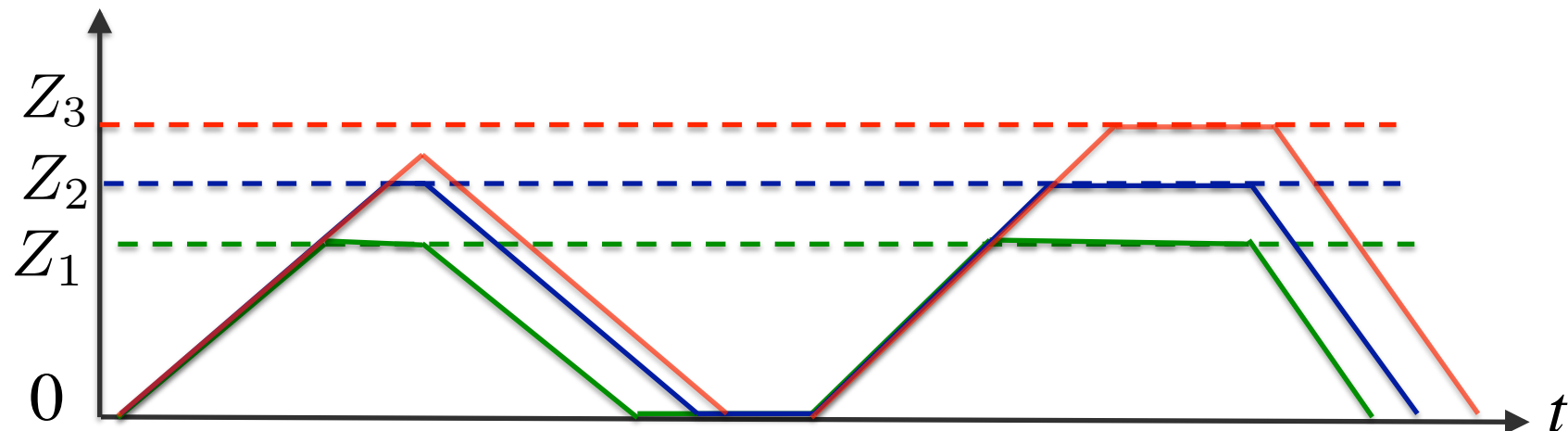
*Less variability
Lower quadratic cost*



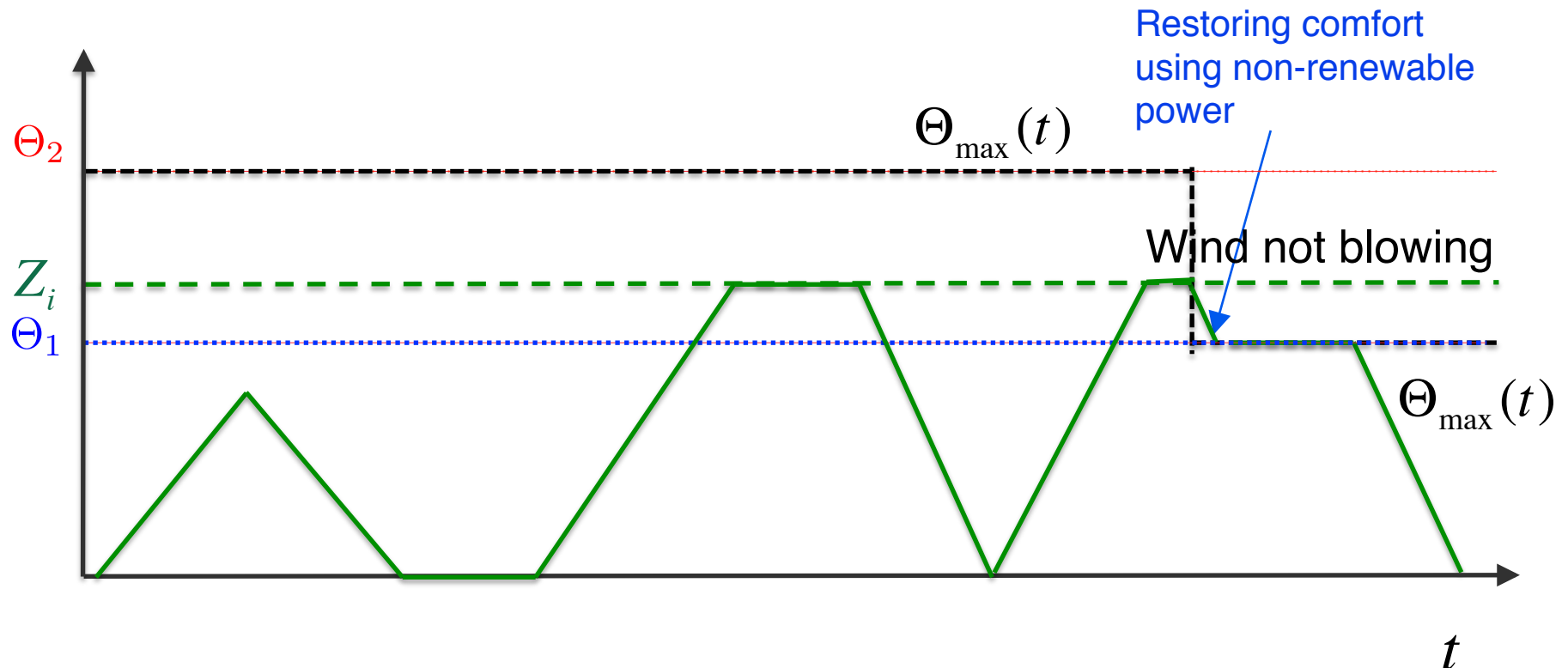
- Lowers operating reserve requirements
- ◆ Impose a quadratic cost on non-renewable power usage $\int P_{\text{non-renewable}}^2(t) dt$

Staggered set-points

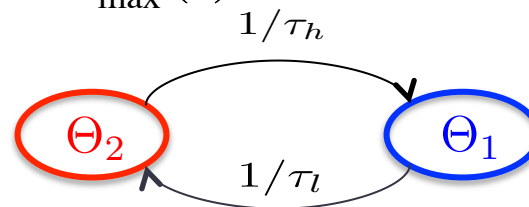
- ◆ De-synchronize load behaviors
- ◆ Choose different set-points (Z_1, Z_2, \dots, Z_N) for different loads



Discomfort: Maximum cooling when comfort range is violated



- ◆ Model changes in $\Theta_{\max}(t)$ as a two state Markov process

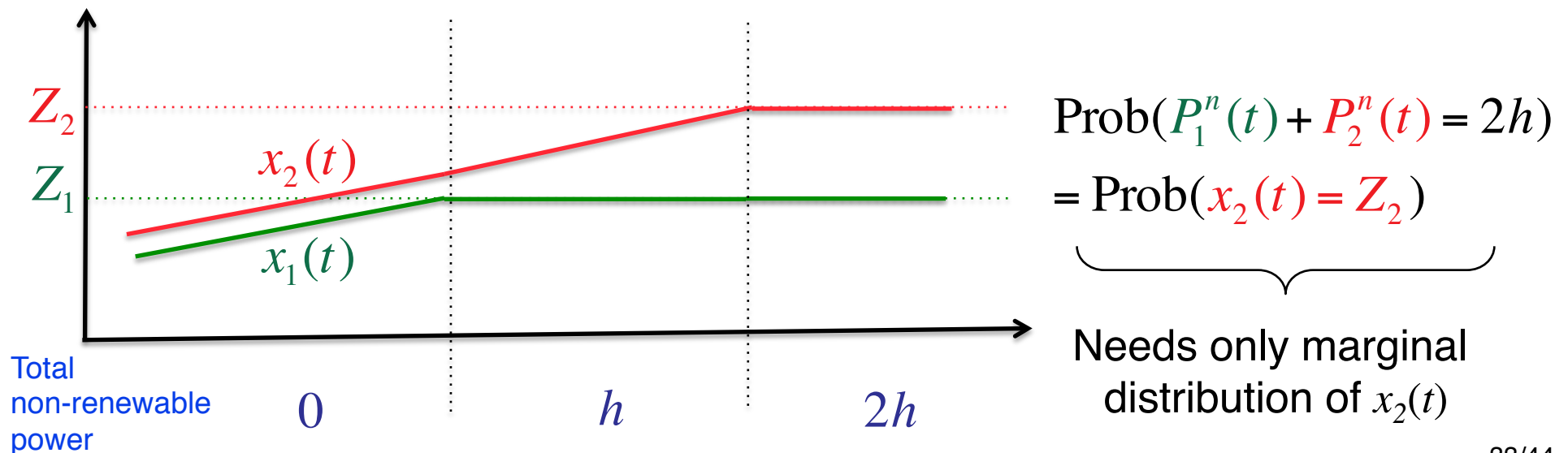


Stochastic optimization problem for $\{Z_1, Z_2, \dots, Z_N\}$

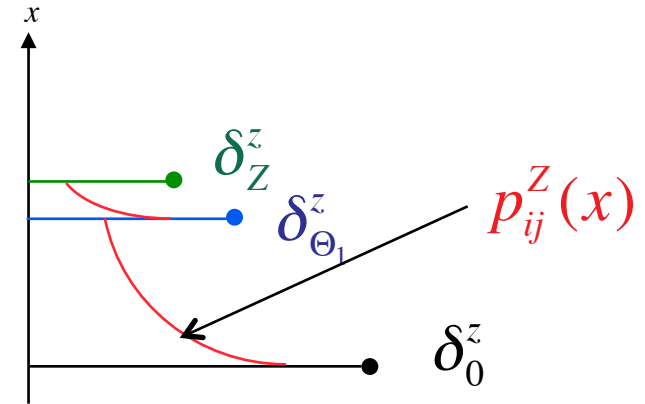
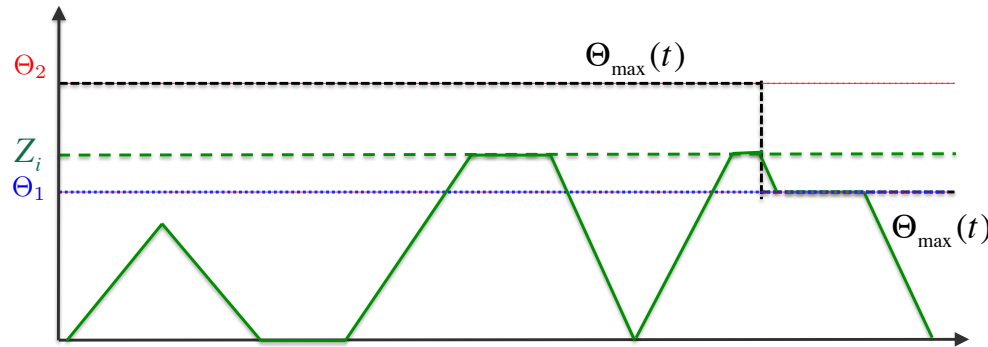
- ◆ Stochastic wind process: $P^w(t)$
- ◆ Temperature dynamics: $\dot{x}_i(t) = h - P_i(t)$
 $P_i(t) = P_i^w(t) + P_i^n(t)$
- ◆ Comfort specification: $\dot{x}_i(t) \in [0, \Theta_{\max}(t)]$
- ◆ Robustness model: Stochastic process $\Theta_{\max}(t)$
- ◆ Set-point control: $P_i^n(t) = \begin{cases} h & \text{if } x_i(t) = \text{Min}(Z_i, \Theta_{\max}(t)) \\ 0 & \text{otherwise} \end{cases}$
- ◆ Cost: $C_N = \lim_{T \rightarrow \infty} \frac{1}{T} \int_0^T \underbrace{\left(P^n(t)\right)^2}_{\text{Variation}} dt + \underbrace{\gamma_N \sum_{i=1}^N \left((x_i(t) - \Theta_{\max}(t))^+\right)^2}_{\text{Discomfort}} dt$

Evaluating the cost: Stochastic coupling

- ◆ Evaluation of cost $\lim_{T \rightarrow \infty} \frac{1}{T} \int_0^T \left(\sum_{i=1}^N P_i^n \right)^2 dx$ is difficult
- ◆ Needs N -dimensional joint probability distribution of temperature states (x_1, x_2, \dots, x_N)
- ◆ Can use stochastic coupling to solve this



The marginal probability distribution of a load



$$\frac{d\mathbf{p}^z(x)}{dx} = \begin{bmatrix} -\frac{q_0+r_0}{k(x)} & \frac{r_1}{k(x)} & \frac{q_1}{k(x)} & 0 \\ \frac{q_0}{h} & -\frac{q_0+r_1}{h} & 0 & \frac{q_1}{h} \\ -\frac{q_0}{c} & 0 & \frac{q_1+r_1}{c} & -\frac{r_1}{c} \\ 0 & -\frac{q_0}{c} & -\frac{r_0}{c} & \frac{q_1+r_1}{c} \end{bmatrix} \mathbf{p}^z(x).$$

where $k(x) = \begin{cases} h & x < \Theta_1 \\ -c & x > \Theta_1 \end{cases}$. The boundary conditions are

$$\begin{bmatrix} h/q_1 & h/q_1 & 0 & 0 \\ 0 & 0 & c/q_1 & c/q_1 \end{bmatrix} \mathbf{p}^z(0) = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \delta_0^z,$$

$$\begin{bmatrix} 0 & 0 & 0 & \frac{h}{q_0+r_0} \\ 0 & 0 & 1 & 0 \\ -\frac{c}{q_0} & 0 & 0 & 0 \\ \frac{h}{r_0} & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \mathbf{p}^z(\Theta_1^-) \\ \mathbf{p}^z(\Theta_1^+) \end{bmatrix} = \begin{bmatrix} \delta_{\Theta_1}^z \\ 0 \\ \delta_{\Theta_1}^z \\ \delta_{\Theta_1}^z \end{bmatrix},$$

$$\begin{bmatrix} \frac{c}{r_1} & 0 & 0 & 0 \\ 0 & \frac{h}{q_0+r_1} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & \frac{c}{q_0} \end{bmatrix} \mathbf{p}^z(z) = \begin{bmatrix} 1 \\ 1 \\ 0 \\ 1 \end{bmatrix} \delta_z^z,$$

$$\int_0^z (\mathbf{1}^T \mathbf{p}^z(x)) dx + \delta_0^z + \delta_z^z + \delta_{\Theta_1}^z = 1,$$

$$\int_0^z p_{01}^z(x) dx + \delta_z^z = \frac{q_1 r_0}{(q_1 + q_0)(r_1 + r_0)}.$$

Marginal probability distribution can be determined through solution of linear system equations

The optimization problem for a finite number of loads

◆ Minimize

$$C^N(Z_1, \dots, Z_N) = \sum (\text{Power level})^2 \times \text{Prob}(\text{Power level}) + \gamma_N \sum \text{Expected Discomfort}$$

◆ Subject to

$$0 \leq Z_1 \leq Z_2 \dots \leq Z_N \leq \Theta_2$$

◆ Difficult

- High dimensional when N is large
- Complex
- Need to solve different problems for different N 's

Continuum limit as $N \rightarrow \infty$.

◆ Solution

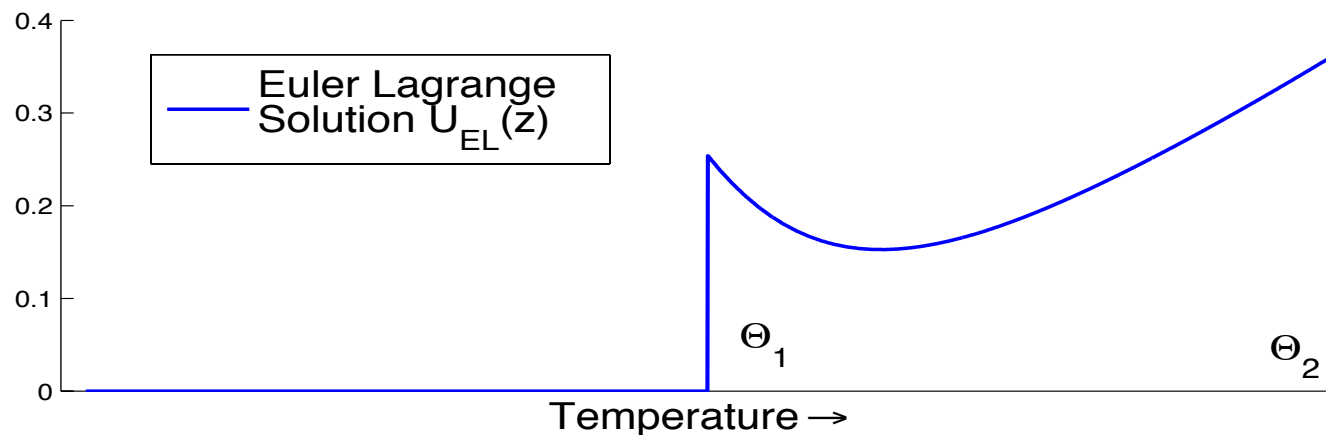
- Study asymptotic limit as $N \rightarrow \infty$.
- Consider Set of loads = $[0,1]$
- Can solve using analytical methods
 - » Pontryagin Minimum Principle
- Solution is explicit!
- Also asymptotic solution is also nearly optimal even for small N !
- Essentially this solves the problem for all N 's

Difficulty with Euler Lagrange method

- ◆ Calculus of variation problem $J[u] = \int_0^{\Theta_2} F(u, u', z) dz$
 - Euler-Lagrange solution

$$u_{EL}(z) = \frac{\gamma \Phi'(z) + 2c(c+h)D_2(z)}{2(h^2 D_1(z) + c^2 D_2(z))}$$

- ◆ This is not an increasing function, and does not satisfy boundary condition



Optimal solution via Pontryagin's minimum principle

◆ Use Pontryagin's Minimum principle

Control $v(z)$

State (non-decreasing): $\frac{d}{dz}u(z) = f(u, v, z) = v^2(z) \geq 0$

Hamiltonian: $H = (u(z) - u_{EL}(z))^2 w(z) + \lambda(z) v^2(z)$

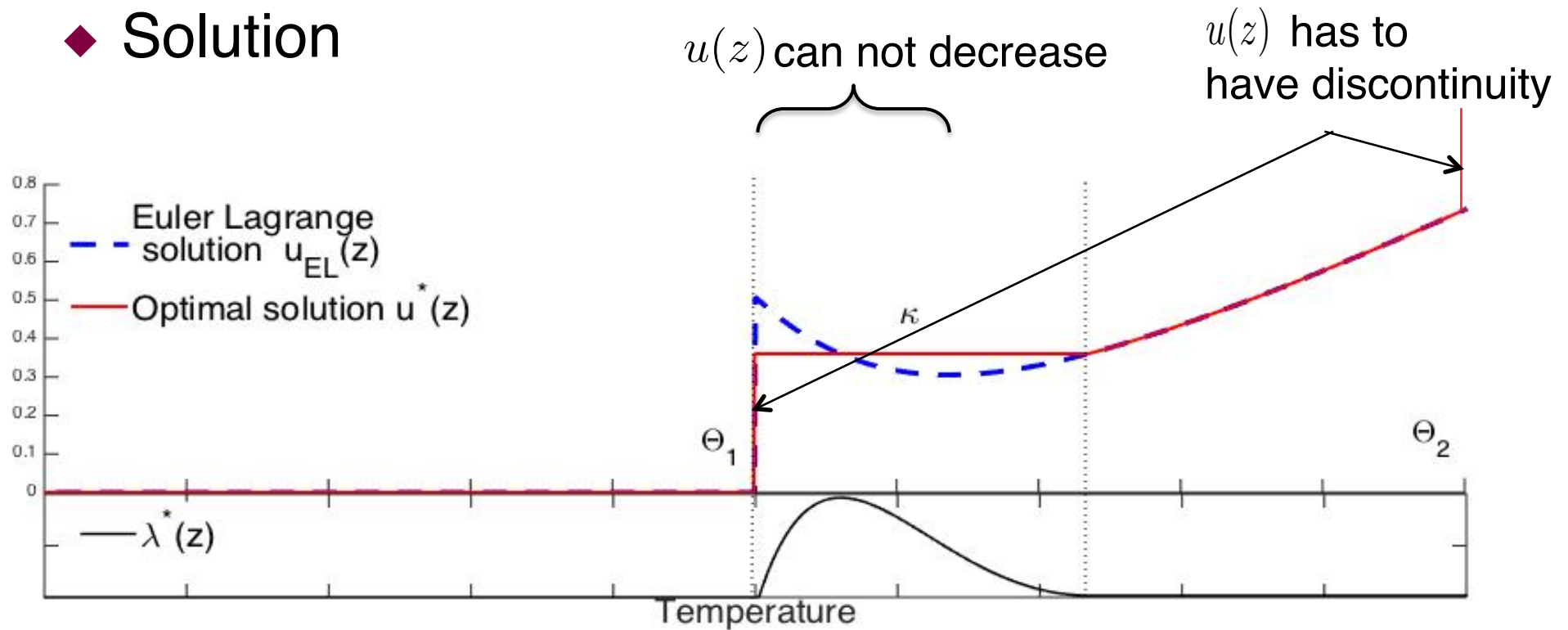
Necessary conditions:

$$\frac{d}{dz}\lambda(z) = -2(u(z) - u_{EL}(z))w(z)$$

$$v(t) = \arg \min_{v \geq 0} [(u(z) - u_{EL}(z))^2 w(z) + \lambda(z) v^2(z)]$$

Optimal solution via Pontryagin's minimum principle

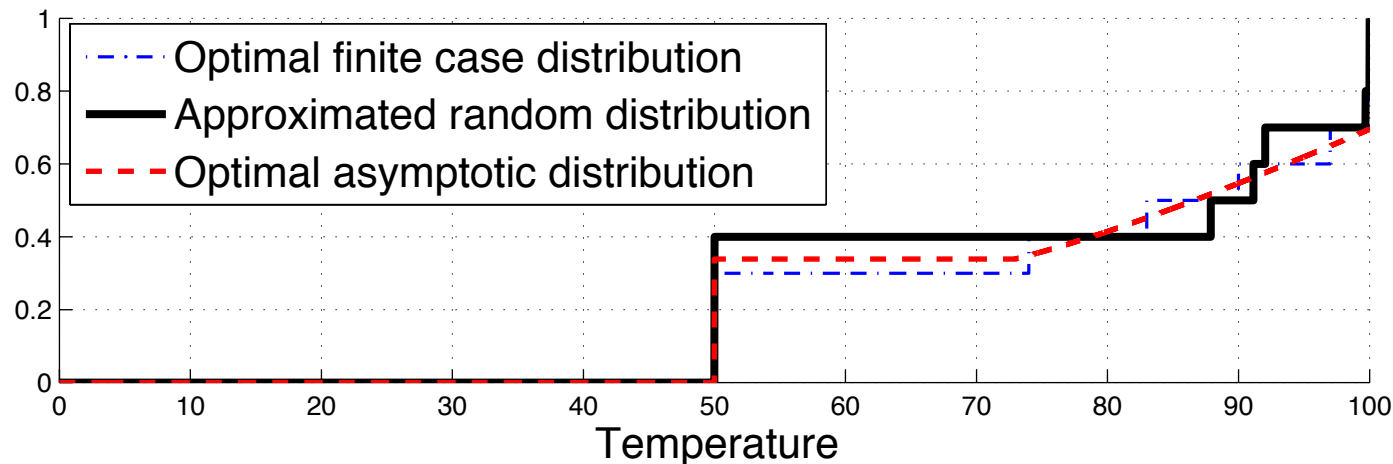
◆ Solution



◆ This gives the optimal staggering of set points

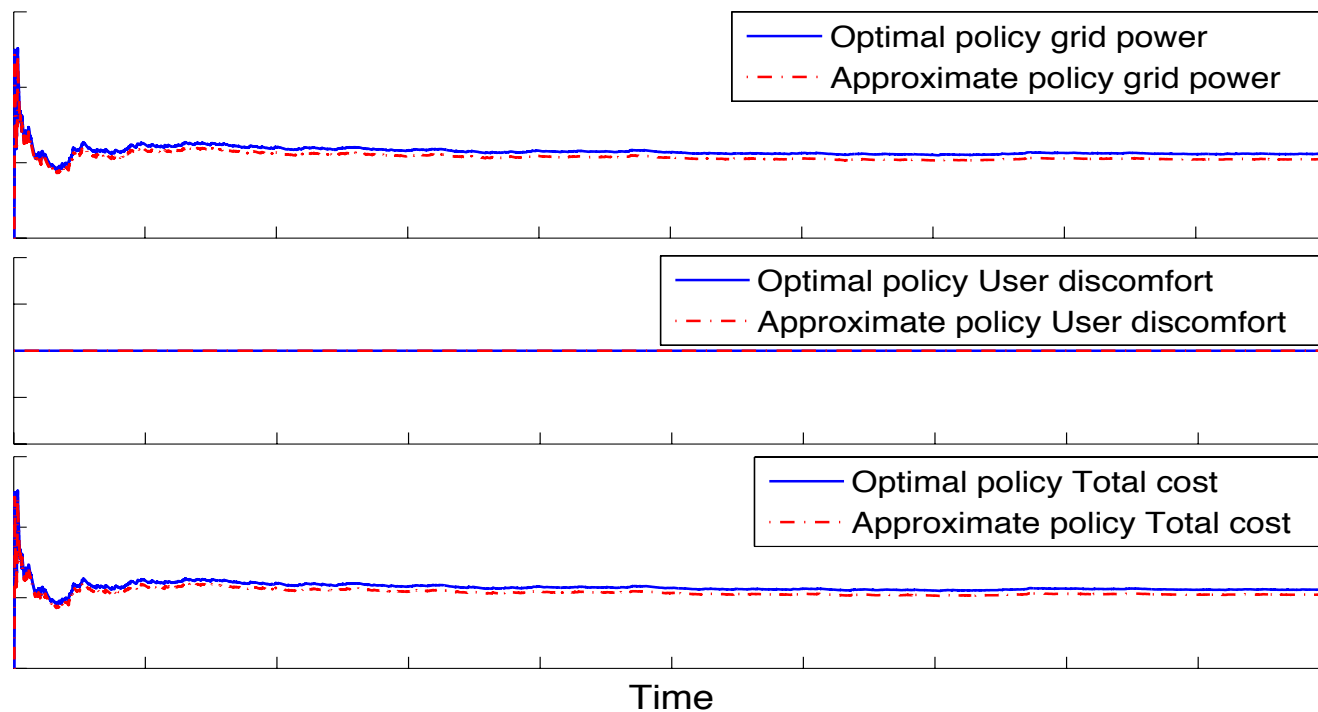
Solving for finite N : Approximation to continuum limit

- ◆ We can generate $\{Z_i\}_1^N$ according to continuum limit distribution, to approximate finite optimal distribution



Some simulation results

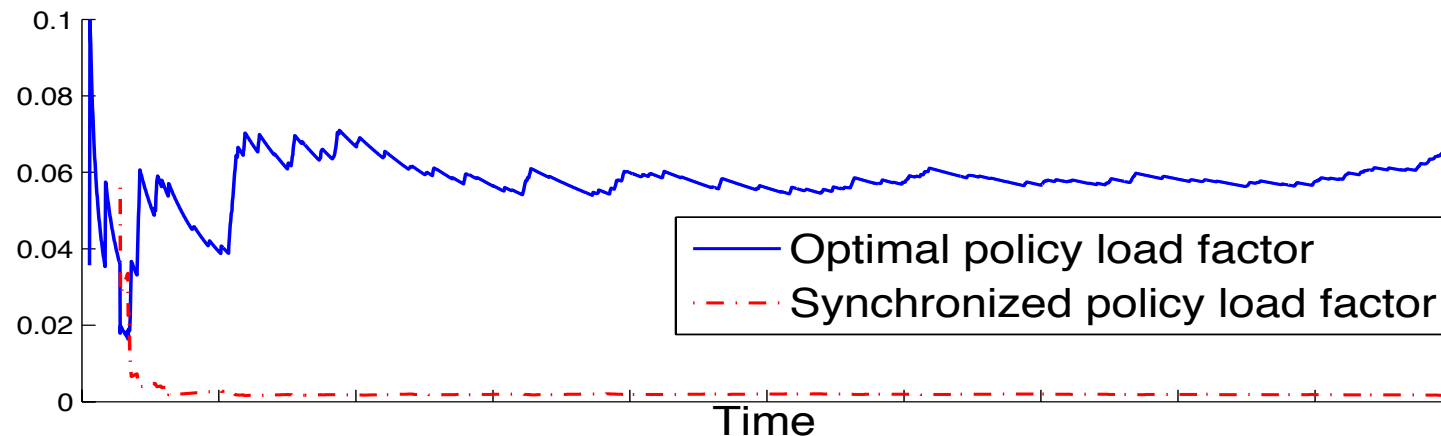
- ◆ The random generation method works reasonably well, even when N is small



Some simulation results - 2

$$\text{Load factor} = \frac{\text{Average power}}{\text{Peak power}}$$

- ◆ Optimal policy has higher load factor than other naive policies



Concluding remarks

- ◆ Design and analysis of an architecture and a simple set-point policy
 - Is architecturally simple to implement
 - De-synchronizes the loads to lower non-renewable peak-to average
 - Alleviates privacy concerns
 - Simple to analyze, low communication requirement, decentralized control
- ◆ Many extensions are feasible
 - Response to comfort variations
 - Availability of wind power
 - Generalize wind model, temperature dynamics, etc.

Thank you